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SUBINTERFACE CRACK IN DISSIMILAR ORTHOTROPIC MATERIALS

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Abstract—A subinterface crack in dissimilar orthotropic materials is considered. The principal axes of one of the materials (say material no. 2 which contains the crack) are aligned along the coordinate axes while the other material's principal axes (say material no. 1) can have an arbitrary angle γ relative to the interface boundary. The effects of material no. 1 on the behavior of material no. 2 are completely through six generalized Dundurs' constants that are functions of γ. When $\gamma = 0$ or when the material constant $\delta^{(1)}$ of material no. 1 is equal to one, two of the six constants vanish and only four left have to be considered. A peculiar phenomenon is observed for $\delta^{(1)} = 1$, i.e. when the material constant $\delta^{(1)}$ of material no. 1 is equal to one the effect of the alignments of material no. 1 on the behavior of material no. 2 is indifferent. Several features of the stress intensity factors for crack faces subjected to self-equilibrating loadings are further investigated without resorting to solving the equations. Some numerical results for crack faces subjected to uniform pressure or shear loadings are also given.

1. INTRODUCTION

The problem of subinterface crack in dissimilar isotropic materials was first investigated by Erdogan (1971). Tamate and Iwasaka (1976) extended Erdogan's work by considering a slant crack near the interface. An asymptotic relationship between interface crack and subinterface crack problems has been derived by Hutchinson *et al.* (1987). Lu and Lardner (1992) recently considered the problem of subinterface cracks in layered isotropic material and provided a comprehensive literature survey of the analysis of isotropic interface or subinterface cracks. The subinterface crack problems for dissimilar anisotropic materials have not been treated as well as the corresponding interface crack problems, which have been widely studied by many researchers (Gotoh (1967), Clements (1971), Willis (1971), Ting (1986), Suo (1990) and Hwu (1993), among others). A crack of arbitrary size and orientation near a bimaterial interface between dissimilar anisotropic materials has been investigated by Miller (1989). Recently, in a series of papers by Wu and Erdogan (1993 a,b,c), the problems of orthotropic laminated plates under membrane and bending loads have been investigated.

In this paper, we shall treat the problem similar to the one investigated by Miller (1989), but from a different point of view. An orthotropic material (material no. 2) with aligned principal axes coinciding with the co-ordinate axes is perfectly bonded to another orthotropic material (material no. 1) with the material principal axes having an arbitrary angle γ relative to the interface boundary. A subinterface crack is situated horizontally in material no. 2. A system of singular integral equations is developed, and explicit expressions for the kernel functions are given in real forms. The kernel functions for many special cases can be recovered from the present results, for example, the kernel functions for the isotropic bimaterial problem are included as special cases in our results. From the explicit forms of the kernel functions, we observe that all material constants of material no. 1 are absorbed by the six generalized Dundurs' constants that are introduced in the paper. Hence the effects of material no. 1 on the behavior of material no. 2 will be completely determined through these six constants. These constants are functions of γ, and only four have to be considered when $\gamma = 0$, since two of them vanish at $\gamma = 0$. Another condition leading to only four generalized constants, which are found to be independent of γ, is when $\delta^{(1)} = 1$. Hence when material constant $\delta^{(1)}$ of material no. 1 is equal to one, the effect of the alignments of material no. 1 on the behavior of material no. 2 is indifferent. The kernel functions developed are further employed to investigate some interesting features for the stress intensity factors when crack faces are subjected to self-equilibrating loadings, which are discussed in Section 4 of this paper. Finally, some numerical results for crack faces subjected to uniform pressure or shear loadings are presented.

2. STROH FORMALISM

A two-dimensional deformation of a linear elastic solid whose field quantities are functions of only x_1 and x_2 is considered here. The general expressions for the displacement **u** and stress function $\boldsymbol{\phi}$ for such a deformation are (Eshelby *et al.* (1953), Stroh (1958)):

$$\mathbf{u} = 2\,\mathrm{Re}\,\{\mathbf{Af(z)}\} \tag{1}$$

$$\boldsymbol{\phi} = 2\,\mathrm{Re}\,\{\mathbf{Bf(z)}\} \tag{2}$$

where Re{ } denotes the real part, and where

$$\mathbf{f(z)} = [f_1(z_1), f_2(z_2), f_3(z_3)]^T \tag{3}$$

with $z_k = x_1 + p_k x_2$ $(k = 1, 2, 3)$. Matrix **A**, with components denoted by a_{kj}, and constants p_k are determined from the following eigenvalue problem:

$$\{c_{i1k1} + p_j(c_{i1k2} + c_{i2k1}) + p_j^2 c_{i2k2}\} a_{kj} = 0 \quad \text{(no sum on } j) \tag{4}$$

where c_{ijkl} are the elastic constants. Without loss of generality, one may take the imaginary part of p_k to be positive. Matrix **B** in eqn (2) is defined by

$$\mathbf{B} = \mathbf{R}^T\mathbf{A} + \mathbf{TAP} \tag{5}$$

where

$$R_{ik} = c_{i1k2} \tag{6}$$

$$T_{ik} = c_{i2k2} \tag{7}$$

and $\mathbf{P} = \mathrm{diag}\,\langle p_1, p_2, p_3\rangle$. The stress function $\boldsymbol{\phi}$ is related to the stress components by

$$(\sigma_{11}, \sigma_{12}, \sigma_{13})^T = \frac{-\partial\boldsymbol{\phi}}{\partial x_2} = -2\,\mathrm{Re}\{\mathbf{BPf'(z)}\} \tag{8}$$

$$(\sigma_{21}, \sigma_{22}, \sigma_{23})^T = \frac{\partial\boldsymbol{\phi}}{\partial x_1} = 2\,\mathrm{Re}\,\{\mathbf{Bf'(z)}\} \tag{9}$$

where

$$\mathbf{f'(z)} = \frac{\mathrm{d}\mathbf{f(z)}}{\mathrm{d}\mathbf{z}} = \left\{\frac{\mathrm{d}f_1(z_1)}{\mathrm{d}z_1}, \frac{\mathrm{d}f_2(z_2)}{\mathrm{d}z_2}, \frac{\mathrm{d}f_3(z_3)}{\mathrm{d}z_3}\right\}^T \tag{10}$$

The matrices **A** and **B** satisfy the following orthogonality relations (Stroh (1958), Chadwick and Smith (1977)):

$$\begin{bmatrix} \mathbf{A} & \bar{\mathbf{A}} \\ \mathbf{B} & \bar{\mathbf{B}} \end{bmatrix} \begin{bmatrix} \mathbf{B} & \bar{\mathbf{B}} \\ \mathbf{A} & \bar{\mathbf{A}} \end{bmatrix}^T = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{11}$$

where **I** is a 3×3 unit matrix. It follows from eqn (11) that the matrix **L** defined as

$$\mathbf{L} = -2\mathrm{i}\mathbf{B}\mathbf{B}^T \tag{12}$$

where $\mathrm{i}^2 = -1$, is real, symmetric, and positive definite, whereas $\mathbf{S}$ given by

$$\mathbf{S} = \mathrm{i}(2\mathbf{A}\mathbf{B}^T - \mathbf{I}) \tag{13}$$

is real. In the following sections, we shall focus on the problem that the assumption that the x_3-axis coincides with the material alignments will always introduce. This makes the anti-plane behavior decouple from that for the in-plane response for orthotropic material. Hence, we shall ignore the anti-plane deformation. Accordingly, the size of the matrices and vectors defined above and in what follows will become 2×2 and 2×1, respectively. For example, the matrices $\mathbf{B}$, $\mathbf{L}$, and $\mathbf{S}$ for orthotropic material are (Dongye and Ting (1989))

$$\mathbf{B} = \begin{bmatrix} -k_1 p_1 C_{66}(C_{12} - p_1^2 C_{22}) & -k_2 p_2 C_{66}(C_{12} - p_2^2 C_{22}) \\ k_1 C_{66}(C_{12} - p_1^2 C_{22}) & k_2 C_{66}(C_{12} - p_2^2 C_{22}) \end{bmatrix} \tag{14}$$

$$\mathbf{L} = \mathrm{diag}\left\langle L_{11} \left(\frac{C_{22}}{C_{11}}\right)^{1/2} L_{11} \right\rangle \tag{15}$$

$$\mathbf{S} = \begin{bmatrix} 0 & -\left(\frac{C_{22}}{C_{11}}\right)^{1/2} S_{21} \\ S_{21} & 0 \end{bmatrix} \tag{16}$$

where

$$S_{21} = \left\{ \frac{C_{66}[(C_{11}C_{22})^{1/2} - C_{12}]}{C_{22}[2C_{66} + C_{12} + (C_{11}C_{22})^{1/2}]} \right\}^{1/2} \tag{17}$$

$$L_{11} = [(C_{11}C_{22})^{1/2} + C_{12}]S_{21} \tag{18}$$

and $C_{\alpha\beta}$ ($\alpha,\beta = 1,2,\ldots,6$) is a contracted notation for c_{ijkl}. The parameters k_1 and k_2 in eqn (14) are complex constants determined by the normalization conditions of eqn (11). The matrices $\mathbf{B}$, $\mathbf{L}$, and $\mathbf{S}$ and constants p_α can be expressed in terms of Krenk's parameters κ, δ, ν, and E (Krenk (1979), Sung and Liou (1994a)) as follows:

$$\mathbf{B} = \begin{cases} \sqrt{\dfrac{E}{8\omega_+\omega_-}} \begin{bmatrix} -[\delta(\omega_+ + \omega_-)]^{1/2} e^{\mathrm{i}\pi/4} & [\delta(\omega_+ - \omega_-)]^{1/2} e^{-\mathrm{i}\pi/4} \\ [\delta(\omega_+ + \omega_-)]^{-1/2} e^{-\mathrm{i}\pi/4} & [\delta(\omega_+ - \omega_-)]^{-1/2} e^{\mathrm{i}\pi/4} \end{bmatrix}, & \kappa > 1 \\ \sqrt{\dfrac{E}{8\omega_-\omega_-}} \begin{bmatrix} [\delta(\omega_- - \mathrm{i}\omega_+)]^{1/2} e^{\mathrm{i}\pi/4} & -[\delta(\omega_- + \mathrm{i}\omega_+)]^{1/2} e^{\mathrm{i}\pi/4} \\ [\delta(\omega_- - \mathrm{i}\omega_+)]^{-1/2} e^{\mathrm{i}\pi/4} & [\delta(\omega_- + \mathrm{i}\omega_+)]^{-1/2} e^{\mathrm{i}\pi/4} \end{bmatrix}, & |\kappa| < 1 \end{cases} \tag{19}$$

$$\mathbf{L} = \frac{E}{2\omega_+}\begin{bmatrix} \delta & 0 \\ 0 & \delta^{-1} \end{bmatrix}, \quad \mathbf{S} = \frac{1-\nu}{2\omega_+}\begin{bmatrix} 0 & -\delta^{-1} \\ \delta & 0 \end{bmatrix} \tag{20}$$

$$p_1 = \begin{cases} \mathrm{i}\delta(\omega_+ + \omega_-), & \kappa > 1 \\ \delta(i\omega_+ - \omega_-), & |\kappa| < 1 \end{cases}, \quad p_2 = \begin{cases} \mathrm{i}\delta(\omega_+ - \omega_-), & \kappa > 1 \\ \delta(i\omega_+ + \omega_-), & |\kappa| < 1 \end{cases} \tag{21}$$

where

$$\omega_+ = \sqrt{(1+\kappa)/2}, \quad \omega_- = \sqrt{|1-\kappa|/2} \tag{22}$$

since the elastic constants are related to Krenk's parameters by

$$C_{11} = \frac{\delta^2 E}{1-\nu^2}, \quad C_{22} = \frac{E}{\delta^2(1-\nu^2)}, \quad C_{12} = \frac{\nu E}{1-\nu^2}, \quad C_{66} = \frac{E}{2(\kappa+\nu)}, \tag{23}$$

where E is the plane-strain effective stiffness, ν the effective Poisson's ratio, δ the stiffness ratio, and κ the shear parameter. The positive definiteness of the strain energy density requires that $\delta > 0$, $E > 0$, $|\nu| < 1$, and $\kappa > -1$. Note that degenerate materials occur when $\kappa = 1$ since $p_1 = p_2$ at $\kappa = 1$. Note also that, for isotropic material, $\delta = \kappa = 1$ and E and ν are reduced to, for the plane-strain case, $E_i/(1-\nu_i^2)$ and $\nu_i/(1-\nu_i)$, respectively, where E_i is the Young's modulus and ν_i is the Poisson's ratio for isotropic material. For later reference, we introduce matrix $\mathbf{E}_j$, defined by

$$\mathbf{E}_j = \mathbf{B}\mathbf{I}_j\mathbf{B}^{-1} \tag{24}$$

where $\mathbf{I}_1 = \mathrm{diag}\langle 1, 0\rangle$, $\mathbf{I}_2 = \mathrm{diag}\langle 0, 1\rangle$, which can also be expressed in terms of Krenk's parameters as

$$\mathbf{E}_1 = \begin{cases} \dfrac{1}{2\omega_-}\begin{bmatrix} \omega_- + \omega_+ & i\delta \\ i\delta^{-1} & \omega_- - \omega_+ \end{bmatrix}, & \kappa > 1 \\[2ex] \dfrac{1}{2\omega_-}\begin{bmatrix} \omega_- - i\omega_+ & \delta \\ \delta^{-1} & \omega_- + i\omega_+ \end{bmatrix}, & |\kappa| < 1 \end{cases}$$

$$\mathbf{E}_2 = \begin{cases} \dfrac{1}{2\omega_-}\begin{bmatrix} \omega_- - \omega_+ & -i\delta \\ -i\delta^{-1} & \omega_- + \omega_+ \end{bmatrix}, & \kappa > 1 \\[2ex] \dfrac{1}{2\omega_-}\begin{bmatrix} \omega_- + i\omega_+ & -\delta \\ -\delta^{-1} & \omega_- - i\omega_+ \end{bmatrix}, & |\kappa| < 1 \end{cases} \tag{25}$$

3. SINGULAR INTEGRAL EQUATIONS

The problem considered is shown in Fig. 1. One aligned orthotropic material (called material no. 2), with the material principal axes parallel with the coordinate axes, is bonded

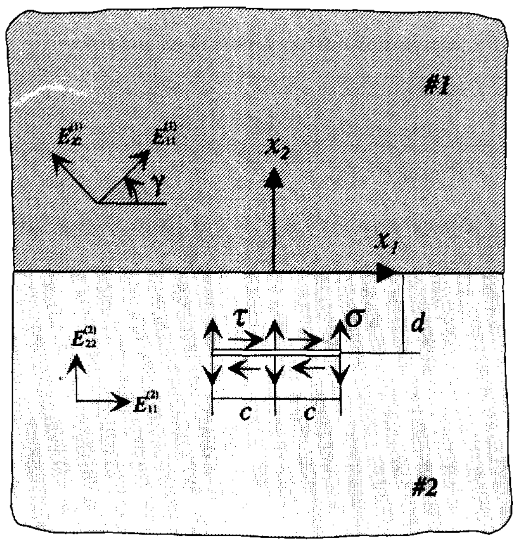

Fig. 1. Geometry of the problem.

perfectly along the interface to another orthotropic material (called material no. 1), whose material in-plane principal axes can have an angle γ relative to the bonded interface. A subinterface crack of length $2c$ is situated a distance d from the interface with material no. 2. The faces of the crack are subjected to self-equilibrating forces. The problem can be simulated by distributing dislocations on the crack faces. By doing so, the problem is then formulated in terms of a system of singular integral equations from which the dislocation densities can be determined. In a recent paper by the present authors (Sung and Liou (1994b)), the problem of two aligned orthotropic materials with cracks being perpendicular to the interface is treated. In that paper, the kernel functions obtained are given explicitly in real forms and four generalized Dundurs' constants are introduced (Sung and Liou (1994b)). In this section, we shall show that for the present problem, even though one of the orthotropic materials is not aligned, the kernel functions can still be expressed in real forms that are useful in investigations of subinterface crack problems. Instead of four constants, this time we shall introduce six generalized Dundurs' constants. The procedures to obtain real kernel functions are similar to those described in the paper mentioned above (Sung and Liou (1994b)). Hence we shall outline in the following only the main results. First, note that the complex forms of the system of singular integral equations and the auxiliary conditions are (Sung and Liou (1994b))

$$\frac{-1}{2\pi}\left\{\int_{-c}^{c}\frac{\mathbf{b}^{(2)}(t)}{t-\xi}\,\mathrm{d}t+\int_{-c}^{c}\mathbf{K}(\xi,t)\mathbf{b}^{(2)}(t)\,\mathrm{d}t\right\}=\mathbf{t}_{\eta}^{-}(\xi),\quad |\xi|<c$$

$$\int_{-c}^{c}\mathbf{b}^{(2)}(t)\,\mathrm{d}t=\mathbf{0}, \tag{26}$$

respectively, where superscript (2) (or (1)) will denote quantities related to material no. 2 (or material no. 1) and

$$\mathbf{K}(\xi,t)=-\sum_{k=1}^{2}\sum_{j=1}^{2}\mathrm{Re}\,\{F_{kj}(\xi,t)\mathbf{E}_k^{(2)}\mathbf{M}\,\overline{\mathbf{E}_j^{(2)}}\} \tag{27}$$

$$\mathbf{b}^{(2)}(t)=\mathbf{L}^{(2)}\mathbf{b}_2(t),\quad |t|\leqslant c \tag{28}$$

In eqns (27) and (28), $\mathbf{b}_2(t)$ are dislocation densities defined on the crack faces, and functions F_{kj} are given by

$$F_{kj}(\xi,t)=[(\xi-t)-d(p_k^{(2)}-\bar{p}_j^{(2)})]^{-1} \tag{29}$$

and matrix $\mathbf{M}$ is given by (Ting (1982, 1992))

$$\mathbf{M}=-\overline{\mathbf{H}}^{-1}\overline{\mathbf{G}} \tag{30}$$

where

$$\mathbf{H}=\Omega(\gamma)^T(\mathrm{i}\mathbf{A}^{(1)}\mathbf{B}^{(1)-1})\Omega(\gamma)+\overline{(\mathrm{i}\mathbf{A}^{(2)}\mathbf{B}^{(2)-1})} \tag{31}$$

and

$$\mathbf{G}=\Omega(\gamma)^T(\mathrm{i}\mathbf{A}^{(1)}\mathbf{B}^{(1)-1})\Omega(\gamma)-(\mathrm{i}\mathbf{A}^{(2)}\mathbf{B}^{(2)-1}) \tag{32}$$

with

$$\mathbf{\Omega}(\gamma) = \begin{bmatrix} \cos(\gamma) & \sin(\gamma) \\ -\sin(\gamma) & \cos(\gamma) \end{bmatrix}. \tag{33}$$

Matrix $\mathbf{M}$ can be expressed explicitly in terms of six generalized Dundurs' constants as follows

$$\mathbf{M} = \frac{1}{1-\beta_1\beta_2-\lambda_1\lambda_2}\begin{bmatrix} \alpha_1+\beta_1\beta_2+\lambda_1\lambda_2 & (\lambda_1-\mathrm{i}\beta_1)(1+\alpha_2) \\ (\lambda_2+\mathrm{i}\beta_2)(1+\alpha_1) & \alpha_2+\beta_1\beta_2+\lambda_1\lambda_2 \end{bmatrix} \tag{34}$$

where the six generalized Dundurs' constants are defined by

$$\begin{aligned} \alpha_1 &= \frac{-U_{11}}{D_{11}}, \quad \alpha_2 = \frac{-U_{22}}{D_{22}}, \\ \beta_1 &= \frac{-W_{12}}{D_{11}}, \quad \beta_2 = \frac{-W_{12}}{D_{22}}, \\ \lambda_1 &= \frac{-D_{12}}{D_{11}}, \quad \lambda_2 = \frac{-D_{12}}{D_{22}} \end{aligned} \tag{35}$$

where $D_{\alpha\alpha}$ and $U_{\alpha\alpha}$ (no sum) are the diagonal components of the matrices $\mathbf{D}$ and $\mathbf{U}$, respectively, and W_{12} and D_{12} are the elements of the first row and second column of $\mathbf{W}$ and $\mathbf{D}$, respectively. Matrices $\mathbf{D}$, $\mathbf{U}$, and $\mathbf{W}$ are given, in terms of Krenk's parameters, by

$$\begin{aligned} \mathbf{D} &= \mathrm{Re}\{\mathbf{H}\} = \mathbf{\Omega}(\gamma)^T\mathbf{L}^{(1)^{-1}}\mathbf{\Omega}(\gamma)+\mathbf{L}^{(2)^{-1}} \\ &= \begin{bmatrix} \dfrac{2\omega_+^{(1)}}{E^{(1)}\delta^{(1)}}(1+\sin^2(\gamma)(\delta^{(1)^2}-1))+\dfrac{2\omega_+^{(2)}}{E^{(2)}\delta^{(2)}} & \dfrac{2\omega_+^{(1)}}{E^{(1)}}\sin(\gamma)\cos(\gamma)(\delta^{(1)^{-1}}-\delta^{(1)}) \\ \dfrac{2\omega_+^{(1)}}{E^{(1)}}\sin(\gamma)\cos(\gamma)(\delta^{(1)^{-1}}-\delta^{(1)}) & \dfrac{2\omega_+^{(1)}\delta^{(1)}}{E^{(1)}}(1+\sin^2(\gamma)(\delta^{(1)^{-2}}-1))+\dfrac{2\omega_+^{(2)}\delta^{(2)}}{E^{(2)}} \end{bmatrix} \end{aligned} \tag{36}$$

$$\begin{aligned} \mathbf{U} &= \mathrm{Re}\{\mathbf{G}\} = \mathbf{\Omega}(\gamma)^T\mathbf{L}^{(1)^{-1}}\mathbf{\Omega}(\gamma)-\mathbf{L}^{(2)^{-1}} \\ &= \begin{bmatrix} \dfrac{2\omega_+^{(1)}}{E^{(1)}\delta^{(1)}}(1+\sin^2(\gamma)(\delta^{(1)^2}-1))-\dfrac{2\omega_+^{(2)}}{E^{(2)}\delta^{(2)}} & \dfrac{2\omega_+^{(1)}}{E^{(1)}}\sin(\gamma)\cos(\gamma)(\delta^{(1)^{-1}}-\delta^{(1)}) \\ \dfrac{2\omega_+^{(1)}}{E^{(1)}}\sin(\gamma)\cos(\gamma)(\delta^{(1)^{-1}}-\delta^{(1)}) & \dfrac{2\omega_+^{(1)}\delta^{(1)}}{E^{(1)}}(1+\sin^2(\gamma)(\delta^{(1)^{-2}}-1))-\dfrac{2\omega_+^{(2)}\delta^{(2)}}{E^{(2)}} \end{bmatrix} \end{aligned} \tag{37}$$

and

$$\begin{aligned} \mathbf{W} &= \mathrm{Im}\{\mathbf{H}\} = \mathrm{Im}\{\mathbf{G}\} = -\mathbf{\Omega}(\gamma)^T\mathbf{S}^{(1)}\mathbf{L}^{(1)^{-1}}\mathbf{\Omega}(\gamma)+\mathbf{S}^{(2)}\mathbf{L}^{(2)^{-1}} \\ &= \begin{bmatrix} 0 & (1-\nu^{(1)})E^{(1)^{-1}}-(1-\nu^{(2)})E^{(2)^{-1}} \\ -(1-\nu^{(1)})E^{(1)^{-1}}+(1-\nu^{(2)})E^{(2)^{-1}} & 0 \end{bmatrix} \end{aligned} \tag{38}$$

Note that $\mathbf{W}$ is independent of γ. Note also that, when material no. 1 is aligned in parallel with the co-ordinate axes, i.e. $\gamma = 0$, matrices $\mathbf{D}$ and $\mathbf{U}$ become diagonal, so that $\lambda_1 = \lambda_2 = 0$. Hence only four generalized Dundurs' constants have to be considered for two aligned orthotropic materials. Another point to be emphasized is that, when material no. 1 has the special property that $\delta^{(1)} = 1$, then the matrices **D** and **U** become diagonal again, and hence only four generalized Dundurs' constants have to be considered for this special case. Furthermore, not only are matrices **D** and **U** diagonal, but matrices **D** and **U** are also independent of γ as long as the condition $\delta^{(1)} = 1$ of material no. 1 is satisfied. The features of $\delta^{(1)} = 1$ of material no. 1 will be further explored later. Note also that, for an isotropic bimaterial problem, the six constants defined are reduced to

$$\alpha_1 = \alpha_2 = \alpha,$$
$$\beta_1 = \beta_2 = \beta,$$
$$\lambda_1 = \lambda_2 = 0 \tag{39}$$

where α and β are Dundurs' constants (1968). On substituting eqns (21), (29), and (34) into eqn (27), the kernel functions become

$$\mathbf{K} = \begin{bmatrix} K_{11} & \delta^{(2)}K_{12} \\ \delta^{(2)^{-1}}K_{21} & K_{22} \end{bmatrix}, \tag{40}$$

where

$$K_{\alpha\beta} = \frac{1}{d(\xi,t)}\sum_{n=0}^{5} c_n^{\alpha\beta}(\xi-t)^n(2\delta^{(2)}d)^{5-n} \tag{41}$$

and $d(\xi, t)$ is given by

$$d(\xi,t) = (2(\xi-t)^2 + 2\omega_+^{(2)^2}(2\delta^{(2)}d)^2)((\xi-t)^4 + (4\omega_+^{(2)^2}-2)(\xi-t)^2(2\delta^{(2)}d)^2 + (2\delta^{(2)}d)^4) \tag{42}$$

The coefficients appearing in eqn (41) are

$$c_0^{11} = 0$$
$$c_1^{11} = \omega_+^{(2)}(N_{12}+N_{21}) - N_{11} - (1+2\omega_+^{(2)^2})N_{22}$$
$$c_2^{11} = (1+2\omega_+^{(2)^2})(\hat{N}_{12}-\hat{N}_{21})$$
$$c_3^{11} = N_{22} + (3-2\omega_+^{(2)^2})N_{11} - 3\omega_+^{(2)}(N_{12}+N_{21})$$
$$c_4^{11} = \hat{N}_{21} - \hat{N}_{12}$$
$$c_5^{11} = -2N_{11}$$

$$c_0^{22} = 2\omega_+^{(2)^2}(\hat{N}_{21}-\hat{N}_{12})$$
$$c_1^{22} = (1+4\omega_+^{(2)^2})\omega_+^{(2)}(N_{12}+N_{21}) - (1+2\omega_+^{(2)^2})N_{11} - (1+8\omega_+^{(2)^4})N_{22}$$
$$c_2^{22} = (4\omega_+^{(2)^2}-1)(\hat{N}_{12}-\hat{N}_{21})$$
$$c_3^{22} = N_{11} - (10\omega_+^{(2)^2}-3)N_{22} + \omega_+^{(2)}(N_{12}+N_{21})$$
$$c_4^{22} = \hat{N}_{12} - \hat{N}_{21}$$
$$c_5^{22} = -2N_{22}$$

$$c_0^{12} = \omega_+^{(2)}(N_{21}-N_{12}) - 2\omega_+^{(2)^2}N_{22}$$
$$c_1^{12} = (2\omega_+^{(2)^2}-1)\hat{N}_{12} - (2\omega_+^{(2)^2}+1)\hat{N}_{21}$$

$$
\begin{aligned}
c_2^{12} &= (2\omega_+^{(2)^2}+1)N_{11}+(4\omega_+^{(2)^2}-1)N_{22}-\omega_+^{(2)}(3N_{21}+(8\omega_+^{(2)^2}-1)N_{12})\\
c_3^{12} &= \hat{N}_{21}-(8\omega_+^{(2)^2}-3)\hat{N}_{12}\\
c_4^{12} &= N_{22}-N_{11}-2\omega_+^{(2)}N_{12}\\
c_5^{12} &= -2\hat{N}_{12}\\
\\
c_0^{21} &= \omega_+^{(2)}(N_{21}-N_{12})+2\omega_+^{(2)^2}N_{22}\\
c_1^{21} &= (2\omega_+^{(2)^2}-1)\hat{N}_{21}-(2\omega_+^{(2)^2}+1)\hat{N}_{12}\\
c_2^{21} &= -(2\omega_+^{(2)^2}+1)N_{11}-(4\omega_+^{(2)^2}-1)N_{22}+\omega_+^{(2)}(3N_{12}+(8\omega_+^{(2)^2}-1)N_{21})\\
c_3^{21} &= \hat{N}_{12}-(8\omega_+^{(2)^2}-3)\hat{N}_{21}\\
c_4^{21} &= N_{11}-N_{22}+2\omega_+^{(2)}N_{21}\\
c_5^{21} &= -2\hat{N}_{21}
\end{aligned}
\tag{43}
$$

In eqns (43), $\hat{N}_{12}$,$\hat{N}_{21}$, and $N_{kj}(k,j = 1,2)$ are defined by

$$
\begin{aligned}
N_{11} &= \frac{\alpha_1+\beta_1\beta_2+\lambda_1\lambda_2}{1-\beta_1\beta_2-\lambda_1\lambda_2} & N_{22} &= \frac{\alpha_2+\beta_1\beta_2+\lambda_1\lambda_2}{1-\beta_1\beta_2-\lambda_1\lambda_2}\\
N_{12} &= \frac{(1+\alpha_2)\beta_1\delta^{(2)^{-1}}}{1-\beta_1\beta_2-\lambda_1\lambda_2} & N_{21} &= \frac{(1+\alpha_1)\beta_2\delta^{(2)}}{1-\beta_1\beta_2-\lambda_1\lambda_2}\\
\hat{N}_{12} &= \frac{(1+\alpha_2)\lambda_1\delta^{(2)^{-1}}}{1-\beta_1\beta_2-\lambda_1\lambda_2} & \hat{N}_{21} &= \frac{(1+\alpha_1)\lambda_2\delta^{(2)}}{1-\beta_1\beta_2-\lambda_1\lambda_2}
\end{aligned}
\tag{44}
$$

where the six generalized Dundurs' constants are

$$
\begin{aligned}
\alpha_1 &= \frac{1-\Lambda\Delta^{-1}[1+\sin^2\gamma(\delta^{(1)^2}-1)]}{1+\Lambda\Delta^{-1}[1+\sin^2\gamma(\delta^{(1)^2}-1)]}\\
\alpha_2 &= \frac{1-\Lambda\Delta[1+\sin^2\gamma(\delta^{(1)^{-2}}-1)]}{1+\Lambda\Delta[1+\sin^2\gamma(\delta^{(1)^{-2}}-1)]}\\
\beta_1 &= \delta^{(2)}\frac{[(1-\nu^{(2)})/E^{(2)}]-[(1-\nu^{(1)})/E^{(1)}]}{(2\omega_+^{(2)}/E^{(2)})+(2\omega_+^{(1)}/E^{(1)})\Delta^{-1}[1+\sin^2\gamma(\delta^{(1)^2}-1)]}\\
\beta_2 &= \delta^{(2)^{-1}}\frac{[(1-\nu^{(2)})/E^{(2)}]-[(1-\nu^{(1)})/E^{(1)}]}{(2\omega_+^{(2)}/E^{(2)})+(2\omega_+^{(1)}/E^{(1)})\Delta[1+\sin^2\gamma(\delta^{(1)^{-2}}-1)]}\\
\lambda_1 &= \delta^{(2)}\frac{\Lambda\sin\gamma\cos\gamma(\delta^{(1)}-\delta^{(1)^{-1}})}{1+\Lambda\Delta^{-1}[1+\sin^2\gamma(\delta^{(1)^2}-1)]}\\
\lambda_2 &= \delta^{(2)^{-1}}\frac{\Lambda\sin\gamma\cos\gamma(\delta^{(1)}-\delta^{(1)^{-1}})}{1+\Lambda\Delta[1+\sin^2\gamma(\delta^{(1)^{-2}}-1)]}
\end{aligned}
\tag{45}
$$

which are found to be determined completely by Krenk's parameters for both orthotropic materials and are functions of γ. In above equations, Λ and Δ are defined by

$$\Lambda = (2\omega_+^{(1)}/E^{(1)})/(2\omega_+^{(2)}/E^{(2)}) \quad \Delta = \delta^{(1)}/\delta^{(2)} \tag{46}$$

It is clear from eqns (40)–(46) that the dependence of the kernel functions on the parameters is given by

$$K_{\alpha\beta} = K_{\alpha\beta}(\xi, t, \kappa^{(2)}, \delta^{(2)}d, \alpha_1, \alpha_2, \delta^{(2)^{-1}}\beta_1, \delta^{(2)}\beta_2, \delta^{(2)^{-1}}\lambda_1, \delta^{(2)}\lambda_2) \tag{47}$$

Since the material constants of material no. 1 are all absorbed by the six generalized Dundurs' constants, and hence the effects of material no. 1 on the behavior of material no. 2 will be directly reflected by these six constants only. Note that these six constants are functions of γ. When $\gamma = 0$, only four constants, i.e. α_1,α_2,β_1, and β_2, have to be considered, since $\lambda_1 = \lambda_2 = 0$ at $\gamma = 0$. Note also that, when material no. 1 has the special property that $\delta^{(1)} = 1$ is satisfied, then one would find that

$$\lambda_1 = \lambda_2 = 0 \tag{48}$$

again, and the other four constants for $\delta^{(1)} = 1$ are independent of γ. The kernel functions for this special material no. 1 are therefore also independent of γ. Hence we can conclude that, when $\delta^{(1)}$ for material no. 1 is equal to unity, then the influence of the alignment of material no. 1 on the behavior of material no. 2 is indifferent. According to eqn (23), one can see immediately that the elastic constants C_{11} and C_{22} of material no. 1 will be related by

$$C_{11} = C_{22} \tag{49}$$

for $\delta^{(1)} = 1$.

The kernel functions for some special cases can be considered from the above general results. First, let us consider the case for an isotropic bimaterial problem, i.e. let $\kappa^{(1)} = \delta^{(1)} = 1$ and $\kappa^{(2)} = \delta^{(2)} = 1$. One finds from eqn (45) that

$$\begin{aligned} \alpha_1 &= \alpha_2 = \alpha \\ \beta_1 &= \beta_2 = \beta \\ \lambda_1 &= \lambda_2 = 0 \end{aligned} \tag{50}$$

which are, of course, all independent of the material angle γ. Substituting these special values in eqn (44), one finds that

$$\begin{aligned} N_{11} &= N_{22} = \frac{\alpha+\beta^2}{1-\beta^2} \\ N_{12} &= N_{21} = \frac{\beta(1+\alpha)}{1-\beta^2} \end{aligned} \tag{51}$$

and

$$\hat{N}_{12} = \hat{N}_{21} = 0.$$

With these results and noting that $\omega_+^{(1)} = \omega_+^{(2)} = 1$, one would find that the kernel functions obtained are the same as those obtained by Erdogan (1971). We next consider the case when $E^{(1)} \ll E^{(2)}$ (or $\Lambda \to \infty$). In this case, $\alpha_1 = \alpha_2 = -1$ and

$$\begin{aligned} N_{11} &= N_{22} = -1, \\ N_{12} &= N_{21} = 0, \\ \hat{N}_{12} &= \hat{N}_{21} = 0. \end{aligned} \tag{52}$$

With these values, one finds that the kernel functions are then reduced to those for the problem of a half-plane solid with a traction-free condition (Sung and Liou (1994a)). If one considers the condition that $E^{(1)} \gg E^{(2)}$ (or $\Lambda \to 0$), then the kernel functions for the problem of a half-plane solid with a clamped boundary can be obtained. This can be achieved by noting that

$$\begin{aligned} \alpha_1 &= \alpha_2 = 1, \\ \delta^{(2)^{-1}}\beta_1 &= \delta^{(2)}\beta_2 = \frac{1-\nu^{(2)}}{2\omega_+^{(2)}}, \\ \lambda_1 &= \lambda_2 = 0 \end{aligned} \tag{53}$$

and

$$\begin{aligned} N_{11} &= N_{22} = \frac{4\omega_+^{(2)^2} + (1-\nu^{(2)})^2}{4\omega_+^{(2)^2} - (1-\nu^{(2)})^2} \\ N_{12} &= N_{21} = \frac{4\omega_+^{(2)}(1-\nu^{(2)})}{4\omega_+^{(2)^2} - (1-\nu^{(2)})^2} \\ \hat{N}_{12} &= \hat{N}_{21} = 0 \end{aligned} \tag{54}$$

when $E^{(1)} \gg E^{(2)}$.

4. STRESS INTENSITY FACTORS

The kernel functions developed in the previous section are used to investigate some phenomena for the stress intensity factors for crack faces subjected to uniform pressure or shear loadings. For crack faces subjected to self-equilibrating loadings, the phenomena discussed below are still valid. Hence we shall discuss only cases for uniform loading. Noting that the explicit forms of the kernel functions given by eqn (40) involve the material parameter $\delta^{(2)}$, it would then be convenient to redefine the unknown function $\mathbf{b}^{(2)}(t)$ by

$$\mathbf{b}^{(2)}(t) = \begin{bmatrix} 1 & 0 \\ 0 & \delta^{(2)^{-1}} \end{bmatrix} \mathbf{b}(t) \tag{55}$$

Hence, the singular integral equations, after the interval of the integration is normalized from $|t| < c$ to $|t| < 1$, become

$$\frac{-1}{2\pi}\left\{\int_{-1}^{1} \frac{\mathbf{b}(t)}{t-\xi}\,\mathrm{d}t + \int_{-1}^{1} \begin{bmatrix} K_{11}^* & K_{12}^* \\ K_{21}^* & K_{22}^* \end{bmatrix} \mathbf{b}(t)\,\mathrm{d}t\right\} = -\begin{bmatrix} 1 & 0 \\ 0 & \delta^{(2)} \end{bmatrix}\begin{Bmatrix} \tau \\ \sigma \end{Bmatrix}, \quad |\xi| < 1 \tag{56}$$

where τ and σ are the applied uniform shear and pressure loadings, respectively. The kernel functions $K_{\alpha\beta}^*$ are obtained by replacing $\delta^{(2)}d$ by $\delta^{(2)}d/c$ in eqns (41) and (42) and are functions of

$$K_{\alpha\beta}^* = K_{\alpha\beta}^*\left(\xi, t, \kappa^{(2)}, \frac{\delta^{(2)}d}{c}, \alpha_1, \alpha_2, \delta^{(2)^{-1}}\beta_1, \delta^{(2)}\beta_2, \delta^{(2)^{-1}}\lambda_1, \delta^{(2)}\lambda_2\right) \tag{57}$$

Hence the solution of $\mathbf{b}(t)$ will have a similar dependence, i.e.

$$\mathbf{b}(t) = 2\begin{bmatrix} f^*_{11} & f^*_{12} \\ f^*_{21} & f^*_{22} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & \delta^{(2)} \end{bmatrix}\begin{Bmatrix} \tau \\ \sigma \end{Bmatrix} \tag{58}$$

where

$$f^*_{\alpha\beta} = f^*_{\alpha\beta}\left(t, \kappa^{(2)}, \frac{\delta^{(2)}d}{c}, \alpha_1, \alpha_2, \delta^{(2)^{-1}}\beta_1, \delta^{(2)}\beta_2, \delta^{(2)^{-1}}\lambda_1, \delta^{(2)}\lambda_2\right) \tag{59}$$

Note that, once the unknown quantities $\mathbf{b}(t)$ have been determined, the stress intensity factors, e.g. at the right crack tip, can be extracted directly by the following formula (Sung and Liou (1994a)):

$$\begin{Bmatrix} K_{II} \\ K_I \end{Bmatrix} = \lim_{\xi \to 1^+} \sqrt{2\pi c(\xi - 1)}\left\{\frac{1}{2\pi}\int_{-1}^{1} \mathbf{b}^{(2)}(t)\frac{dt}{\xi - t}\right\}$$
$$= \sqrt{\pi c}\begin{bmatrix} f_{11} & \delta^{(2)}f_{12} \\ \delta^{(2)^{-1}}f_{21} & f_{22} \end{bmatrix}\begin{Bmatrix} \tau \\ \sigma \end{Bmatrix} \tag{60}$$

where the functions $f_{\alpha\beta}$ are defined by

$$f_{\alpha\beta}\left(\kappa^{(2)}, \frac{\delta^{(2)}d}{c}, \alpha_1, \alpha_2, \delta^{(2)^{-1}}\beta_1, \delta^{(2)}\beta_2, \delta^{(2)^{-1}}\lambda_1, \delta^{(2)}\lambda_2\right)$$
$$= \frac{1}{\pi}\lim_{\xi \to 1^+} \sqrt{2(\xi - 1)}\int_{-1}^{1} f^*_{\alpha\beta}\left(t, \kappa^{(2)}, \frac{\delta^{(2)}d}{c}, \alpha_1, \alpha_2, \delta^{(2)^{-1}}\beta_1, \delta^{(2)}\beta_2, \delta^{(2)^{-1}}\lambda_1, \delta^{(2)}\lambda_2\right)\frac{dt}{\xi - t}$$
$$= \hat{f}_{\alpha\beta}\left(t = 1, \kappa^{(2)}, \frac{\delta^{(2)}d}{c}, \alpha_1, \alpha_2, \delta^{(2)^{-1}}\beta_1, \delta^{(2)}\beta_2, \delta^{(2)^{-1}}\lambda_1, \delta^{(2)}\lambda_2\right) \tag{61}$$

Functions $\hat{f}_{\alpha\beta}$, which are regular in the interval $|t| \leqslant 1$, are related to functions $f^*_{\alpha\beta}$ by

$$\hat{f}_{\alpha\beta}\left(t, \kappa^{(2)}, \frac{\delta^{(2)}d}{c}, \alpha_1, \alpha_2, \delta^{(2)^{-1}}\beta_1, \delta^{(2)}\beta_2, \delta^{(2)^{-1}}\lambda_1, \delta^{(2)}\lambda_2\right)$$
$$= \sqrt{1 - t^2}\, f^*_{\alpha\beta}\left(t, \kappa^{(2)}, \frac{\delta^{(2)}d}{c}, \alpha_1, \alpha_2, \delta^{(2)^{-1}}\beta_1, \delta^{(2)}\beta_2, \delta^{(2)^{-1}}\lambda_1, \delta^{(2)}\lambda_2\right) \tag{62}$$

We have to mention that, for the isotropic bimaterial problem, the counterpart of eqn (60) is

$$\begin{Bmatrix} K_{II} \\ K_I \end{Bmatrix} = \sqrt{\pi c}\begin{bmatrix} \tilde{f}_{11} & \tilde{f}_{12} \\ \tilde{f}_{21} & \tilde{f}_{22} \end{bmatrix}\begin{Bmatrix} \tau \\ \sigma \end{Bmatrix} \tag{63}$$

where

$$\tilde{f}_{\alpha\beta} = \tilde{f}_{\alpha\beta}\left(\frac{d}{c}, \alpha, \beta\right) \tag{64}$$

are functions that usually need to be calculated numerically. In the following, we investigate some interesting phenomena for the stress intensity factors for the case when the alignments of material no. 1 are along the coordinate axes, i.e. $\gamma = 0$. It is noted again that $\lambda_1 = \lambda_2 = 0$ when $\gamma = 0$, and hence only four generalized Dundurs' constants will be involved in the following discussions.

Case I

Suppose material no. 2 has the special property that $\kappa^{(2)} = 1$; the kernel functions $K^*_{\alpha\beta}$ in eqn (57) will then be exactly the same as those functions for the isotropic bimaterial problem as long as the generalized Dundurs' constants for orthotropic bimaterials are chosen such that

$$\alpha_1 = \alpha_2 = \alpha \quad \text{(note that, if } \alpha_1 = \alpha_2\text{, then } \delta^{(1)} = \delta^{(2)}\text{)}$$

$$\delta^{(2)^{-1}}\beta_1 = \delta^{(2)}\beta_2 = \beta \tag{65}$$

are satisfied where α,β are Dundurs' constants. Hence, from eqns (61) and (64), we can conclude that

$$f_{\alpha\beta}\left(\frac{\delta^{(2)}d}{c},\alpha,\beta\right) = \tilde{f}_{\alpha\beta}\left(\frac{d}{c},\alpha,\beta\right) \tag{66}$$

This implies that the solutions to the orthotropic bimaterial problem with material compositions satisfying the above conditions can be obtained directly from those for the isotropic bimaterial problem.

Case II

Suppose that the compositions of two orthotropic materials are such that $\mathbf{W} = \mathbf{0}$; the kernel functions will then totally vanish if the conditions

$$\alpha_1 = \alpha_2 = 0 \tag{67}$$

are also satisfied. This implies that the stress intensity factors for such dissimilar orthotropic compositions will be totally independent of the material constants. The results are therefore the same as those for a homogeneous medium.

Case III

Suppose that the material axes are rotated by 90° for both orthotropic materials. One would then find that the parameters $\delta^{(1)}$ and $\delta^{(2)}$ are changed in the following way:

$$\delta^{(1)} \to \delta^{(1)^{-1}},$$

$$\delta^{(2)} \to \delta^{(2)^{-1}} \tag{68}$$

Hence the constants α_1, α_2, $\delta^{(2)^{-1}}\beta_1$, and $\delta^{(2)}\beta_2$ appearing in the functions $f_{\alpha\beta}$ will remain unchanged if $\delta^{(1)} = \delta^{(2)}$ is satisfied. This is due to the fact that

$$\alpha_1 = \alpha_2 = \frac{(2\omega_+^{(2)}/E^{(2)}) - (2\omega_+^{(1)}/E^{(1)})}{(2\omega_+^{(2)}/E^{(2)}) + (2\omega_+^{(1)}/E^{(1)})}$$

$$\delta^{(2)^{-1}}\beta_1 = \delta^{(2)}\beta_2 = \frac{(1-\nu^{(2)})/E^{(2)} - (1-\nu^{(1)})/E^{(1)}}{(2\omega_+^{(2)}/E^{(2)}) + (2\omega_+^{(1)}/E^{(1)})} \tag{69}$$

when $\delta^{(1)} = \delta^{(2)}$. Hence the functions $f_{\alpha\beta}$ for the 90°-rotated case can be obtained by just replacing $\delta^{(2)}d/c$ by $\delta^{(2)^{-1}}d/c$ from the before-rotated situation. With these newly obtained

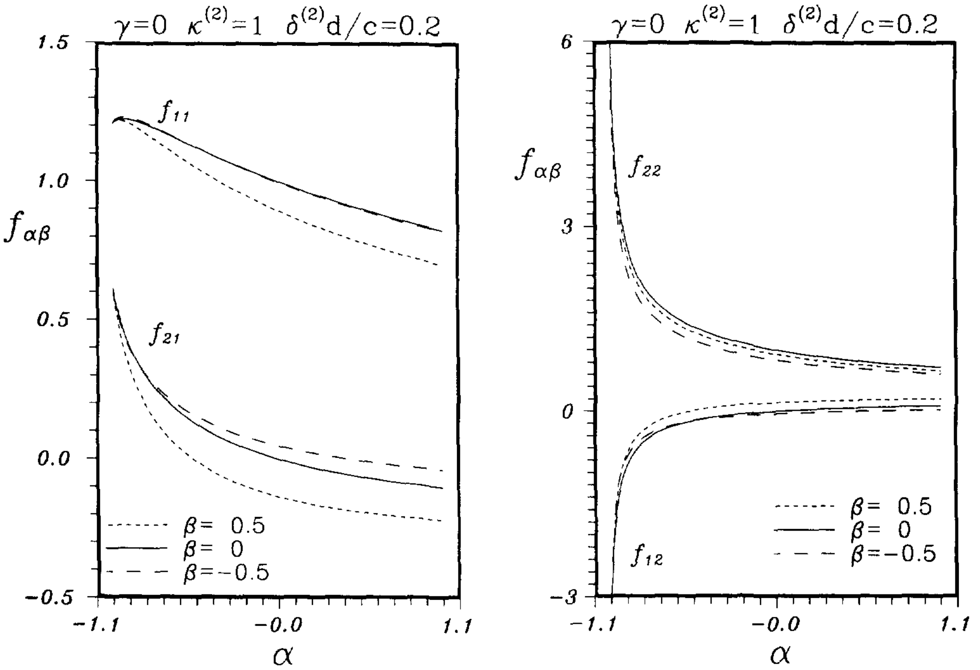

Fig. 2 (a) and (b). Functions $f_{\alpha\beta}$ defined in eqn (66) versus α for $\gamma = 0$, $\kappa^{(2)} = 1$, and $\delta^{(2)}d/c = 0.2$ and for various values of β.

functions, the stress intensity factors for the 90°-rotated case problem can then be computed directly by eqn (60), with $\delta^{(2)}$ replaced by $\delta^{(2)^{-1}}$ in that formula.

5. SOME NUMERICAL RESULTS

(A) $\gamma = 0$

As has been discussed in the previous section, when the alignment of the principal axes of material no. 1 is along the coordinate axes, i.e. $\gamma = 0$, and if $\kappa^{(2)} = 1$, then the functions $f_{\alpha\beta}$, related to the stress intensity factors given by eqn (60), can be obtained from the corresponding isotropic bimaterial problem. Since, to our knowledge, no information is available for $f_{\alpha\beta}$ in the literature even for the isotropic bimaterial case, we therefore compute these functions numerically. A numerical scheme based on that of Gerasoulis (1982) is developed. Figures 2(a) and 2(b) are the results for these functions versus α for $\delta^{(2)}d/c = 0.2$. The effects of β are also shown in these figures. One should note that material no. 1 becomes very flexible when $\alpha_1 = \alpha_2 \to -1$ and material no. 1 becomes very stiff when $\alpha_1 = \alpha_2 \to 1$. Note also that, when $\alpha = \beta = 0$, the problem becomes homogeneous. Another point that we wish to address is to see the effect of $\kappa^{(2)}$ on functions $f_{\alpha\beta}$ for two aligned orthotropic materials, i.e. $\gamma = 0$. For simplicity, only the results for $\alpha_1 = \alpha_2 = \alpha_0$ (i.e. $\delta^{(1)} = \delta^{(2)}$) and $\delta^{(2)^{-1}}\beta_1 = \delta^{(2)}\beta_2 = \beta_0 = 0$ are given. The results are shown in Figs 3(a) and 3(b). It is observed that f_{22} and f_{12} (Fig. 3(b)) are insensitive to the parameter $\kappa^{(2)}$ for this special choice of the four generalized Dundurs' constants.

(B) The effect of γ

As noted previously, the generalized Dundurs' constants are functions of γ. It would therefore be useful to plot these constants versus γ to see how dependent they are. Note that α_1,α_2,λ_1, and λ_2 are dependent on the material parameters $\delta^{(1)}$,$\delta^{(2)}$, and Λ only. Hence we plot in Figs 4(a) and 4(b) only the behavior of α_1,α_2,λ_1, and λ_2 for $\delta^{(1)} = 2$ and $\delta^{(2)} = 1$ for various values of Λ. Note that $\alpha_1 = \alpha_2 = 1$ and $\lambda_1 = \lambda_2 = 0$ when $\Lambda = 0$ (corresponding to material no. 1 being very stiff) and $\alpha_1 = \alpha_2 = -1$ when $\Lambda \to \infty$ (corresponding to material no. 1 being very flexible). It is observed from eqn (45) that constants α_1,α_2 and λ_1,λ_2 are

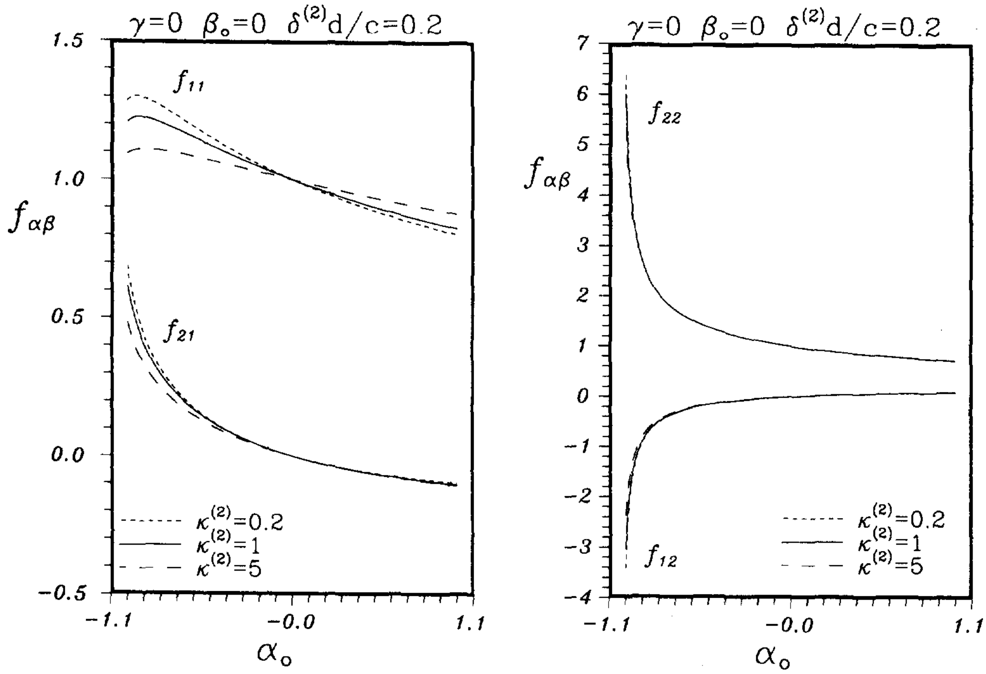

Fig. 3 (a) and (b). Functions $f_{\alpha\beta}$ defined in eqn (61) versus α_0 for $\gamma = 0, \beta_0 = 0$, and $\delta^{(2)}d/c = 0.2$ and for various values of $\kappa^{(2)}$.

related to each other by

$$\alpha_1(\gamma) = \alpha_2\left(\frac{\pi}{2} - \gamma\right),$$

$$\lambda_1(\gamma) = \lambda_2\left(\frac{\pi}{2} - \gamma\right) \tag{70}$$

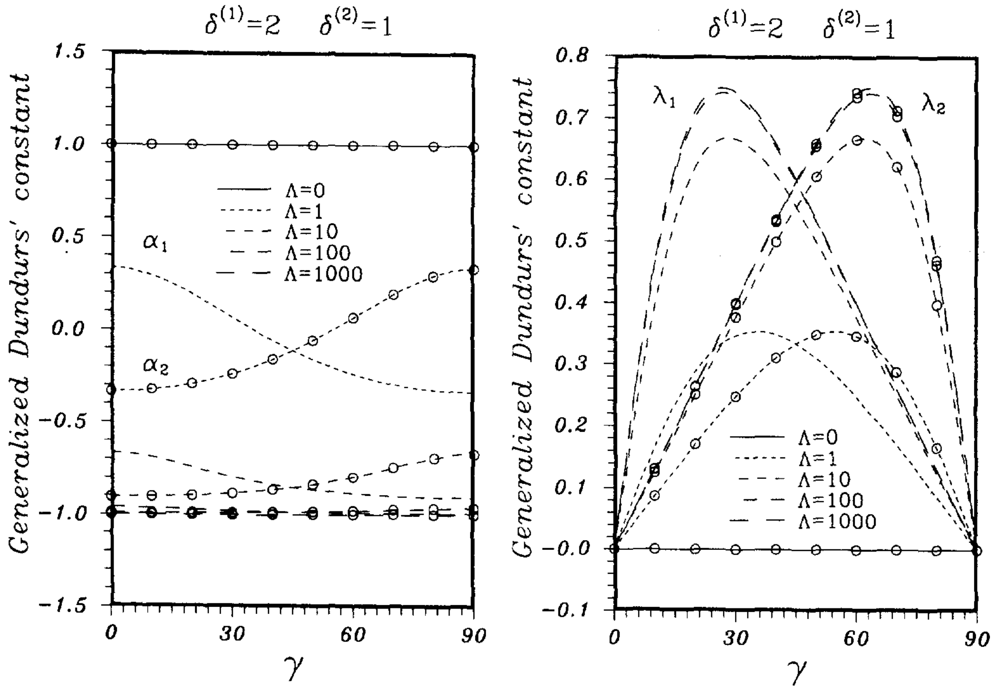

Fig. 4 (a) and (b). Generalized Dundurs' constants $\alpha_1, \alpha_2, \lambda_1$, and λ_2 versus γ for $\delta^{(1)} = 2$, and $\delta^{(2)} = 1$ and for various values of Λ.

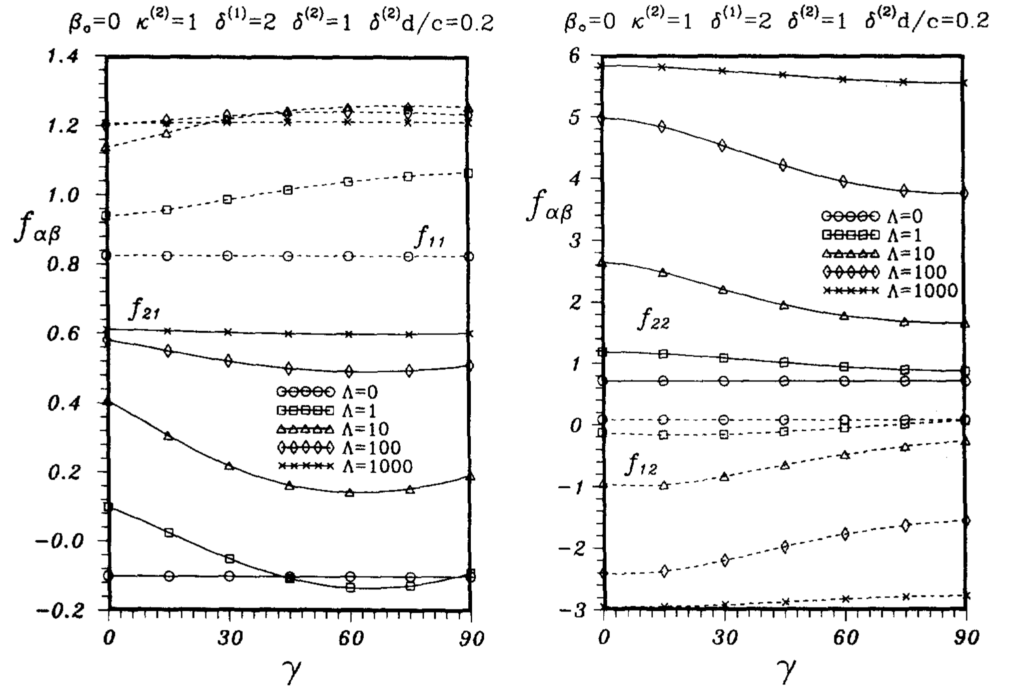

Fig. 5 (a) and (b). Functions $f_{\alpha\beta}$ defined in eqn (61) versus γ for $\beta_0 = 0$, $\kappa^{(2)} = 1$, $\delta^{(1)} = 2$, $\delta^{(2)} = 1$, and $\delta^{(2)}d/c = 0.2$ and for various values of Λ.

as long as $\delta^{(2)} = 1$. Figures 4(a) and 4(b) have this feature since $\delta^{(2)} = 1$ in plotting these figures. It is noted from Fig. 4 that the results for α_1 and α_2 (and also for λ_1 and λ_2) for $\Lambda = 100$ and $\Lambda = 1000$ are quite close.

To see the effect of γ on the functions defined in eqn (61), we computed these functions numerically for the following special material compositions:

Material no. 1	Material no. 2	
$\delta^{(1)} = 2$	Case I:	Case II:
(*orthotropic*)	$\kappa^{(2)} = 1$	$\kappa^{(2)} = 5$
	$\delta^{(2)} = 1$	$\delta^{(2)} = 1$
	(*isotropic*)	(*orthotropic*)

and, for simplicity, we let

$$\frac{1-\nu^{(1)}}{E^{(1)}} = \frac{1-\nu^{(2)}}{E^{(2)}} \tag{71}$$

so that $\delta^{(2)^{-1}}\beta_1 = \delta^{(2)}\beta_2 = \beta_0 = 0$. Note that the effect of parameter $\kappa^{(1)}$ of material no. 1 is reflected by the parameter Λ. The depth of the crack is chosen such that $\delta^{(2)}d/c = 0.2$. Figures 5(a) and 5(b) are the results for $f_{\alpha\beta}$ for the case when material no. 2 is isotropic. The results for the case when material no. 2 is orthotropic are plotted in Figs 6(a) and 6(b). It is found from the numerical results that

$$\begin{aligned} f_{21}(\kappa^{(2)} = 1) &\approx f_{21}(\kappa^{(2)} = 5) \quad \text{for} \quad \Lambda = 0 \\ f_{22}(\kappa^{(2)} = 1) &\approx f_{22}(\kappa^{(2)} = 5) \quad \text{for} \quad \Lambda = 0, 1, 10 \\ f_{12}(\kappa^{(2)} = 1) &\approx f_{12}(\kappa^{(2)} = 5) \quad \text{for} \quad \Lambda = 0, 1 \end{aligned} \tag{72}$$

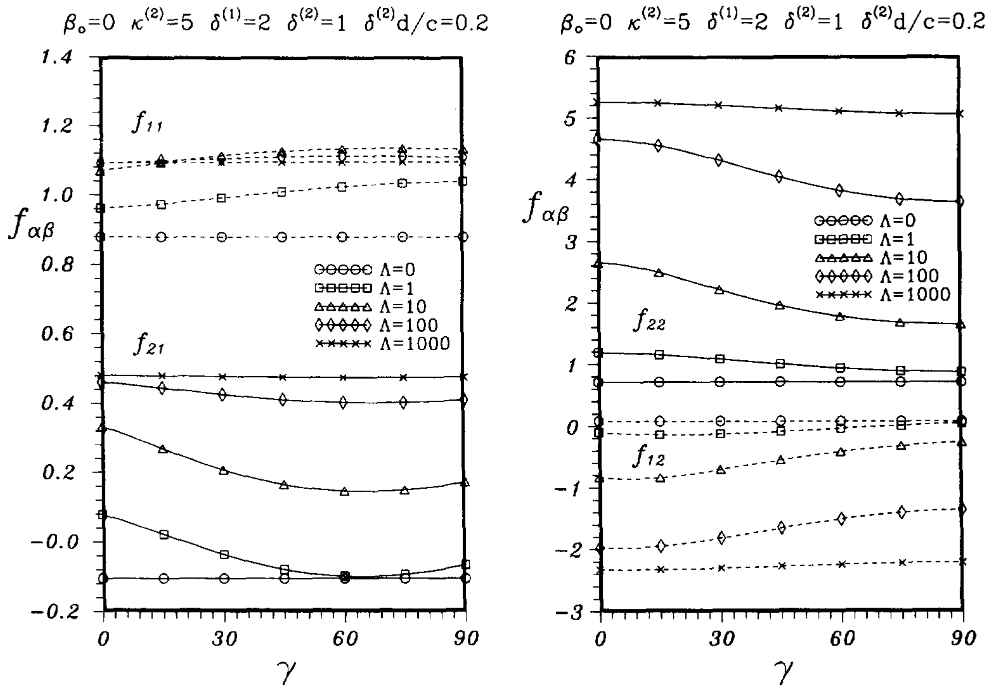

Fig. 6 (a) and (b). Functions $f_{\alpha\beta}$ defined in eqn (61) versus γ for $\beta_0 = 0$, $\kappa^{(2)} = 5$, $\delta^{(1)} = 2$, $\delta^{(2)} = 1$, and $\delta^{(2)}d/c = 0.2$ and for various values of Λ.

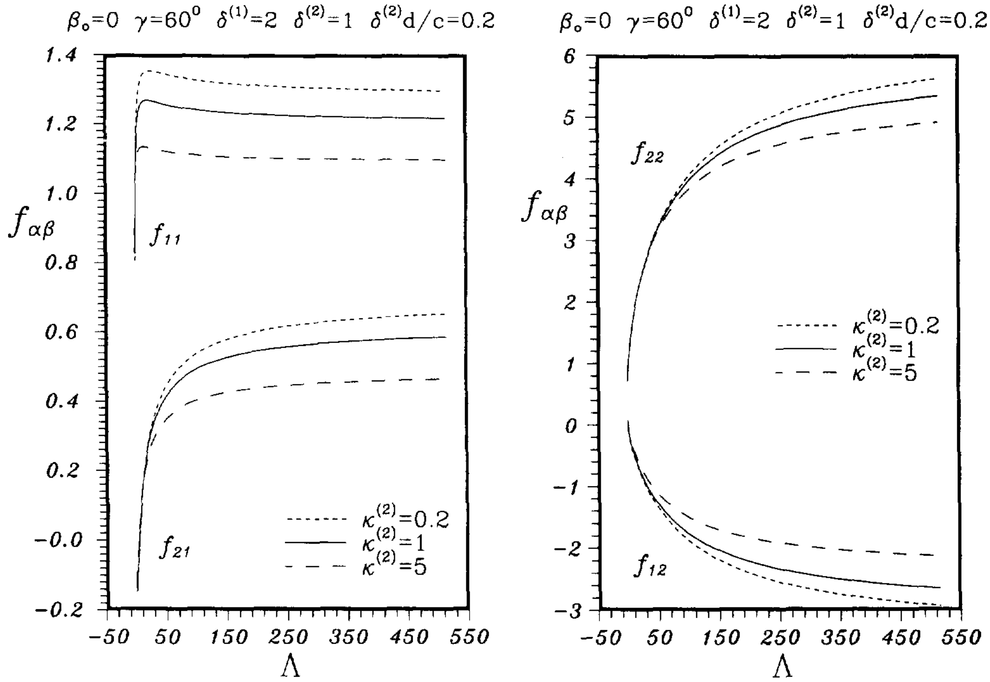

Fig. 7 (a) and (b). Functions $f_{\alpha\beta}$ defined in eqn (61) versus Λ for $\beta_0 = 0$, $\gamma = 60^0$, $\delta^{(1)} = 2$, $\delta^{(2)} = 1$, and $\delta^{(2)}d/c = 0.2$ and for various values of $\kappa^{(2)}$.

for all γ. To see the effect of Λ on these functions, we therefore plot $f_{\alpha\beta}$ versus Λ in Figs 7(a) and 7(b) for various values of $\kappa^{(2)}$ for $\gamma = 60^0$. It may be seen from these figures that functions f_{22} and f_{12} are insensitive to $\kappa^{(2)}$ when Λ is small, whereas function f_{11} increases rapidly for small values of Λ and then becomes approximately constant for the other values of Λ.

6. CONCLUDING REMARKS

The problem of a subinterface crack in dissimilar orthotropic materials is investigated. With the use of the material parameters introduced by Krenk in conjunction with the Stroh formalism, the kernels of the singular integral equations are expressed in real forms. These forms reveal that the effects of the material (without crack) on the behavior of the material (with crack) are determined completely through six generalized Dundurs' constants. The alignments of the material (without crack) are found to have no effect on the behavior of the material (with crack) when the material (without crack) has the property that $\delta = 1$. The real expressions for the kernels are further employed to discuss some features of the stress intensity factors. Some numerical results for the stress intensity factors for crack faces subjected to uniform pressure or shear loadings are also given.

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